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Impact of the honk effect on the stability of traffic flow

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ABSTRACT

In this paper, we propose an extended car-following model which takes into account the honk effect. The analytical and numerical results show that the honk effect improves the stability of traffic flow. The dependence of the stability on the properties of the honk effect is investigated in this paper.

1. Introduction

Over the past couple of decades, many traffic flow models have been proposed to study various complex traffic phenomena [1–18]. However, these models cannot directly describe the impacts of the honk effect on traffic flow. On the other hand, in some countries such as China, a driver often honks his horn if the speed of the leading vehicle is too low. Once a driver hears the horn, he may immediately change lanes or accelerate his vehicle depending on the traffic state. In order to study the honk effect, Jia et al. [19] developed an extended cellular automaton model for two-lane traffic flow and obtained some important results, but they did not explore the impacts of the honk effect on the car-following behavior.

Recently, noting that the driver of a vehicle (bicycle) will honk his horn to prevent a bicycle (pedestrian) from running on his lane in the mixed traffic system with vehicles, bicycles and pedestrians, Tang et al. [20] proposed a new bicycle-following model and a new pedestrian-following model with honk effects and their study shows that honk effects can significantly enhance the bicycle and pedestrian flows. However, they did not further explore the impacts of the honk effect on the car-following behavior. Tang et al. [21] thus recently developed an extended optimal velocity (OV) model with the honk effect and the authors found that the honk effect enhances the equilibrium velocity and flow when the traffic density is moderate while it has little impact on the equilibrium velocity and flow when the density is very low or very high. To our knowledge, although a lot of results have been achieved, the impacts of the honk effect on the stability of traffic flow have not been investigated. In this paper, we thus develop a new car-following model based on the model in Ref. [21] and use it to explore the impacts of the honk effect on the stability of traffic flow.

2. Car-following model with the honk effect

Among the existing car-following models, the optimal velocity model [3] is the simplest one, i.e.

$$\frac{dv_n(t)}{dt} = \frac{V(\Delta x_n(t)) - v_n(t)}{\tau}, \tag{1}$$

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where $v_n(t)$, $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ are respectively the nth vehicle's velocity, headway at t, $V(\Delta x_n(t))$ is the nth vehicle's optimal velocity that is determined by its headway, and τ is the delay time. To further investigate the phenomena of traffic flow, some extended OV models [4–6] were proposed within the general form of

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \frac{V(\Delta x_n(t), \Delta x_{n+1}(t), \ldots, \Delta x_{n+m}(t)) - v_n(t)}{\tau}, \tag{2}$$

where $\Delta x_{n+i}(t) = x_{n+i+1}(t) - x_{n+i}(t)$ and $V(\Delta x_n(t), \Delta x_{n+1}(t), \ldots, \Delta x_{n+m}(t))$ is the nth vehicle's optimal velocity that is determined by the headways $\Delta x_{n+i}(t) = x_{n+i+1}(t) - x_{n+i}(t)$. Besides the classical OV model and its extension, there are many other car-following models with one or multiple relative velocities [7–11], which can be expressed as the following generalized equation:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f(v_n, \Delta x_n, \Delta v_n, \Delta v_{n+1}, \ldots, \Delta v_{n+k}), \tag{3}$$

where $\Delta v_{n+i}(t) = v_{n+i+1}(t) - v_{n+i}(t)$. It has been proved that the inclusion of relative velocities in the model can improve the stability of traffic flow. Zhao and Gao [12] then found that the FVD (full velocity difference) model [8] may produce collision under some given conditions, and thus they proposed a car-following model taking into account the leading vehicle's acceleration, i.e.

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f\left(v_n, \Delta x_n, \Delta v_n, \frac{\mathrm{d}v_{n+1}}{\mathrm{d}t}\right). \tag{4}$$

There are some other car-following models such as those in Ref. [13–18]. However, the above mentioned models cannot directly describe the impacts of the honk effect on traffic flow. On the other hand, in many countries (e.g. China), the drivers will honk their horn if their leading vehicle's velocity is less than theirs. When a driver hears the horn, he/she will accelerate or change lanes depending on the traffic state at that time. To study the honk effect, Tang et al. [21] developed an extended OV model with the honk effect, but the authors assumed that drivers will consider the honk effect only when the density is moderate. In fact, the driver must consider the honk effect during the whole following process since his following vehicle may honk the horn at any moment. In a real traffic system, the honk effect is complex and dependent on many factors (the current vehicle's velocity, maximum velocity, headway, its following vehicle's headway, the relative velocity between the current and preceding vehicles, etc.), so we should apply many observed data to define the exact function of the honk effect. However, the honk effect will depend on two main factors, i.e. the desire that the following vehicle honks the horn and the anticipation influence of the honk effect on the current vehicle, where the desire is often determined by the following vehicle's headway and the anticipation influence is often determined the current vehicle's velocity. This shows that the honk effect is often determined by Δx_{n-1} and $v_n(t)$. Thus, the extended OV model [21] can be rewritten as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \frac{V(\Delta x_n(t)) - v_n}{\tau} + \mu g(\Delta x_{n-1}, v_n(t)), \tag{5}$$

where $g(\Delta x_{n-1}, v_n(t))$ is the honk effect resulting from the $(n-1)$th vehicle, and μ is the honk effect coefficient.

We next define the honk effect $g(\Delta x_{n-1}, v_n(t))$. The vehicle may honk the horn at any moment in many developing countries (e.g. China), so the driver should consider the honk effect during the whole following process. This shows that the honk effect will have impacts on the driver no matter whether his/her following driver honks the horn or not, so we can assume that $g(\Delta x_{n-1}, v_n)$ is irrelevant to the desire that the $(n-1)$th vehicle driver honks the horn (i.e. $g(\Delta x_{n-1}, v_n)$ is irrelevant to Δx_{n-1}). However, the driver must adjust his/her acceleration based on his/her current velocity though he/she considers the honk effect, i.e. the honk effect will be restricted by his/her current velocity. This illustrates that the honk effect of the $(n-1)$th vehicle on its leading vehicle will be determined the nth vehicle's velocity. Thus we can for simplicity define the honk effect as follows:

$$g(\Delta x_{n-1}, v_n) = \frac{v_{\max} - v_n}{\tau'}, \tag{6}$$

where τ' is the reaction time that it takes for the nth car to adjust its acceleration based on the honk effect. Eq. (6) indicates that the honk effect will be magnified with the increase of the density. In fact, the anticipation impacts resulted by the honk effect increase with the density because the difference between the maximum velocity and current velocity increases with the density, but the real impact of the honk effect on traffic flow will decrease with the density because the space that the driver can accelerate decreases with the density. Thus, the impacts of the honk effect on traffic flow can be adjusted by applying the honk effect coefficient μ. However, the parameter μ has little qualitative impacts on the following analytical and numerical results, so we use Eq. (6) to explore the impacts of the honk effect on traffic flow. Thus Eq. (5) can be rewritten as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \frac{V(\Delta x_n) - v_n}{\tau} + \mu\frac{v_{\max} - v_n}{\tau'}.\text{[1]} \tag{7}$$

[1] Note: we will in the future use many experimental data to calibrate a more exact honk effect function and further study the impacts of the honk effect on traffic flow.

There are many different methods for discretizing Eq. (7), but the difference in discretizing schemes have little qualitative impacts on the numerical results. Thus, we here use the asymmetric forward difference to discretize Eq. (7), i.e.

$$\Delta x_n(t+2\tau) = \Delta x_n(t+\tau) + \tau\left[V\left(\Delta x_{n+1}(t)\right) - V\left(\Delta x_n(t)\right)\right] + \mu\tau\left[g\left(v_{n+1}\right) - g\left(v_n\right)\right]. \tag{8}$$

Combining with Eq. (7), Eq. (9) can be rewritten as follows:

$$\Delta x_n(t+2\tau) = \Delta x_n(t+\tau) + \tau\left[V\left(\Delta x_{n+1}(t)\right) - V\left(\Delta x_n(t)\right)\right] + \frac{\mu}{\tau'}\left(-\Delta x_n(t+\tau) + \Delta x_n(t)\right). \tag{9}$$

In addition, here we define the optimal velocity $V(\Delta x_n)$ of Eq. (5) as follows [3]:

$$V(\Delta x_n) = \frac{v_{\max}}{2}\left[\tanh\left(\Delta x_n - h_c\right) + \tanh(h_c)\right], \tag{10}$$

where $v_{\max}$ is the maximum velocity and h_c refers to the safe distance.

3. Linear stability analysis

In this section, we study the linear stability of Eq. (10). The solution of the uniformly steady state of Eq. (10) can be written as follows:

$$x_{n,0}(t) = hn + V(h)t, \quad h = L/N, \tag{11}$$

where N is the number of cars, L is the length of the road and h is the average headway.

Let $y_n(t)$ be a small deviation from the uniform solution $x_{n,0}(t)$, $x_n(t) = x_{n,0}(t) + y_n(t)$, then, a linear equation can be obtained from Eq. (10),

$$\Delta y_n(t+2\tau) = \Delta y_n(t+\tau) + \tau V'(h)\left[\Delta y_{n+1}(t) - \Delta y_n(t)\right] + \frac{\mu}{\tau'}\left[-\Delta y_n(t+\tau) + \Delta y_n(t)\right], \tag{12}$$

where $V'(h)$ is the derivative of the optimal velocity $V(\Delta x)$ at $\Delta x = h$.

Let $\Delta y_n(t) = A\exp(ikn + zt)$, Eq. (12) can be rewritten as follows:

$$e^{2z\tau} - e^{z\tau} - \tau V'(h)\left[e^{ik} - 1\right] - \frac{\mu}{\tau'}\left[-e^{z\tau} + 1\right] = 0 \tag{13}$$

Solving Eq. (13) with respect to z, we can find that the leading term of z is of the order of ik. Here $z \to 0$ when $ik \to \infty$, and so z can be expressed by a long wave as $z = z_1(ik) + z_2(ik)^2 + \cdots$. Substituting it into Eq. (13) and neglecting the terms with order greater than 2, the two roots of z are obtained

$$z_1 = \frac{V'(h)}{1+\mu/\tau'}, \qquad z_2 = \frac{V'(h) - \left(3+\mu/\tau'\right)\tau z_1^2}{2\left(1+\mu/\tau'\right)}, \tag{14}$$

It is clear that the flow becomes unstable if $z_2 < 0$ and stable if $z_2 > 0$. Thus, the demarcation point between stable and unstable conditions (hereafter called the neutral stability point) is

$$\alpha = \frac{1}{\tau} = \frac{\left(3+\mu/\tau'\right)V'(h)}{\left(1+\mu/\tau'\right)^2}, \tag{15}$$

where $\alpha = \frac{1}{\tau}$ is the reactive coefficient. If

$$\alpha < \frac{\left(3+\mu/\tau'\right)V'(h)}{\left(1+\mu/\tau'\right)^2}, \tag{16}$$

then an unstable flow will evolve from a small perturbation into uniform flow. We find that the neutral stability curve will gradually be pulled down with the increase of the parameter $\frac{\mu}{\tau'}$, which shows that the honk effect can improve the stability of traffic flow (see Fig. 1).

For comparison purposes, the neutral stability curves given by the optimal velocity model [3] is also depicted in Fig. 1. Since $V'(h)$ reaches the maximum value $0.5v_{\max}$ at the turning point $h = h_c$, the critical point (h_c, α_c) exists for these neutral stability curves. Fig. 1 shows that with the same value h_c, the critical value α_c and the neutral stability curve obtained from our model is lower than the ones of the optimal velocity model [3] and the critical value α_c and the neutral curve will drop with the increase of the parameter $\frac{\mu}{\tau'}$. This shows that the honk effect can improve the stability of traffic flow and the stable region will increase as the parameter $\frac{\mu}{\tau'}$ increases.

In order to further explore the impacts that the honk effect has on the stability of traffic flow, we next investigate the relationship between the critical value α_c and the honk effect coefficient μ (the reactive time τ'). Combining with Eq. (6), we can obtain the critical point α_c, i.e.

$$\alpha_c = \frac{\left(3+\mu/\tau'\right)v_{\max}}{2\left(1+\mu/\tau'\right)^2}. \tag{17}$$

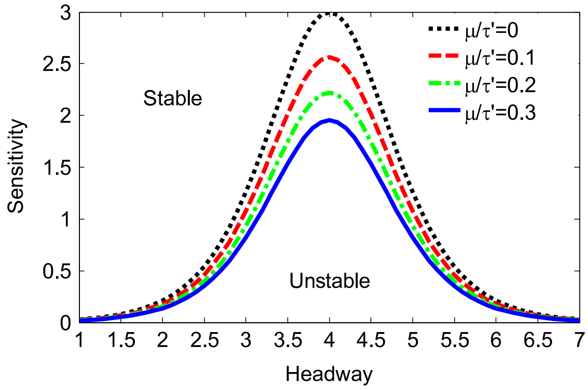


Fig. 1. Phase diagram on the headway-sensitivity $(\Delta x, \alpha)$ plane, where each pair of parameters (μ, τ') has one neutral stability curve with a maximum sensitivity. The neutral stability curve divides the space into stable and unstable regions.

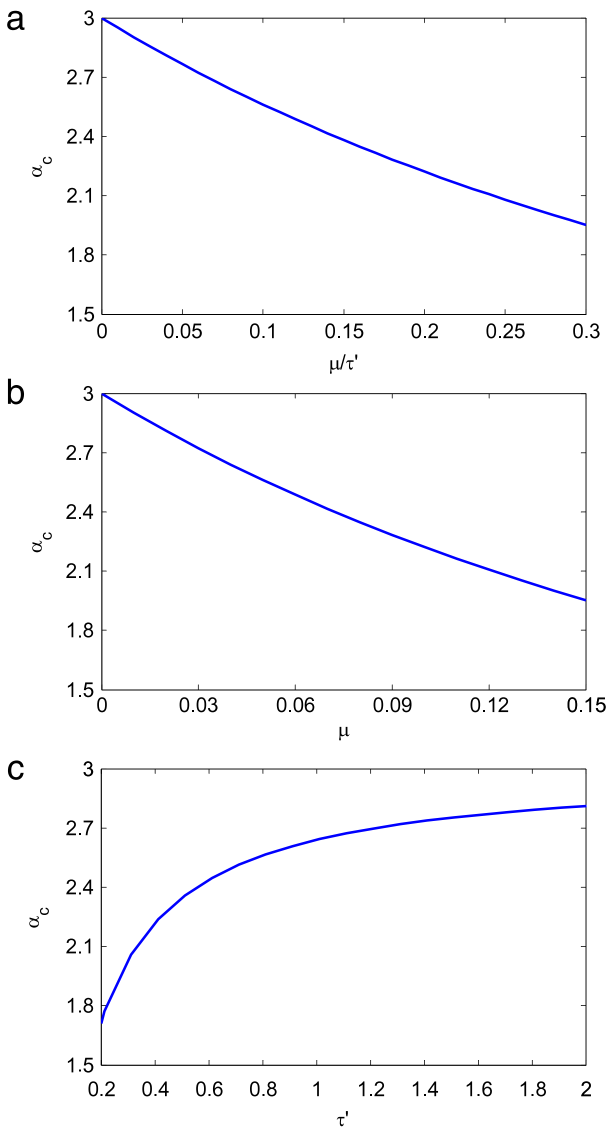
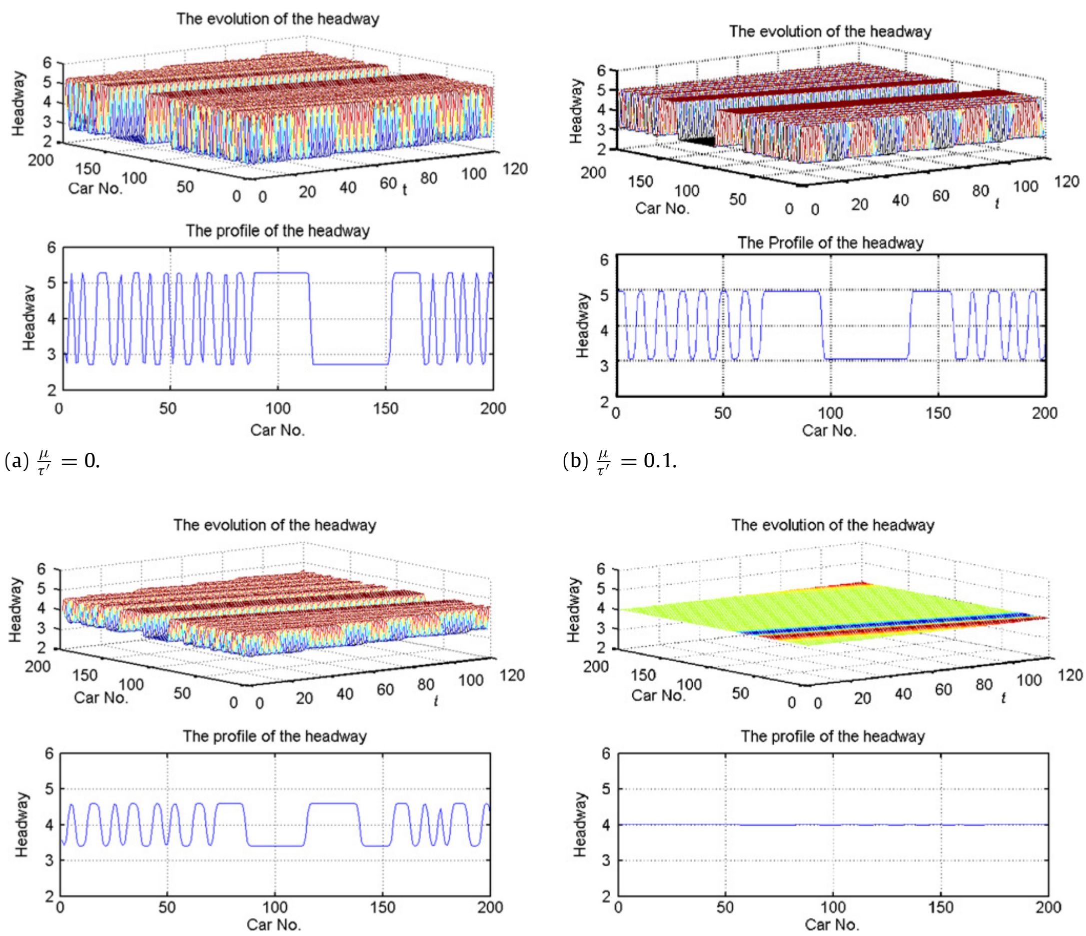


Fig. 2. The relationship between the critical value α_c and the parameters $\frac{\mu}{\tau'}$, μ and τ', where the parameter τ' is fixed at 0.5 in Fig. 2(b) and the parameter μ is fixed at 0.1 in Fig. 2(c).

Set $F(\mu, \tau') = \frac{(3+\mu/\tau')}{(1+\mu/\tau')^2}$, it is very easy to prove that

$$\frac{\partial F(\mu, \tau')}{\partial \mu} = -\frac{(5 + \mu/\tau')}{\tau'(1 + \mu/\tau')^3} < 0, \tag{18}$$



(a) $\frac{\mu}{\tau'} = 0$.

(b) $\frac{\mu}{\tau'} = 0.1$.

(c) $\frac{\mu}{\tau'} = 0.2$.

(d) $\frac{\mu}{\tau'} = 0.3$.

Fig. 3. Headway evolution after 10^4 time steps and the profile at $t = 10^4$, where (a) is the output of the OV model and (b)–(d) are the outputs from our model under different values of the parameter $\frac{\mu}{\tau'}$.

$$\frac{\partial F\left(\mu, \tau'\right)}{\partial \tau'} = \frac{\left(5 + \mu/\tau'\right)\mu}{(\tau')^2\,(1 + \mu/\tau')^3} > 0. \tag{19}$$

Thus, we obtain that the critical value α_c decreases as the honk effect coefficient μ increases, and increases as the reaction time τ' increases, which further illustrates that the stability of traffic flow will increase with the honk effect coefficient μ and decrease with the reactive time τ' (see Fig. 2).

4. Simulation

In this section, we use numerical tests to study the impacts of the honk effect on the stability of traffic flow. The initial values of the model parameters in the simulation are given below:

$$\begin{cases} \Delta x_n(0) = \Delta x_n(1) = \Delta x_0, & n \neq 0.5N,\ n \neq 0.5N + 1 \\ \Delta x_n(0) = \Delta x_n(1) = \Delta x_0 + 0.1, & n = 0.5N \\ \Delta x_n(0) = \Delta x_n(1) = \Delta x_0 - 0.1, & n = 0.5N + 1, \end{cases} \tag{20}$$

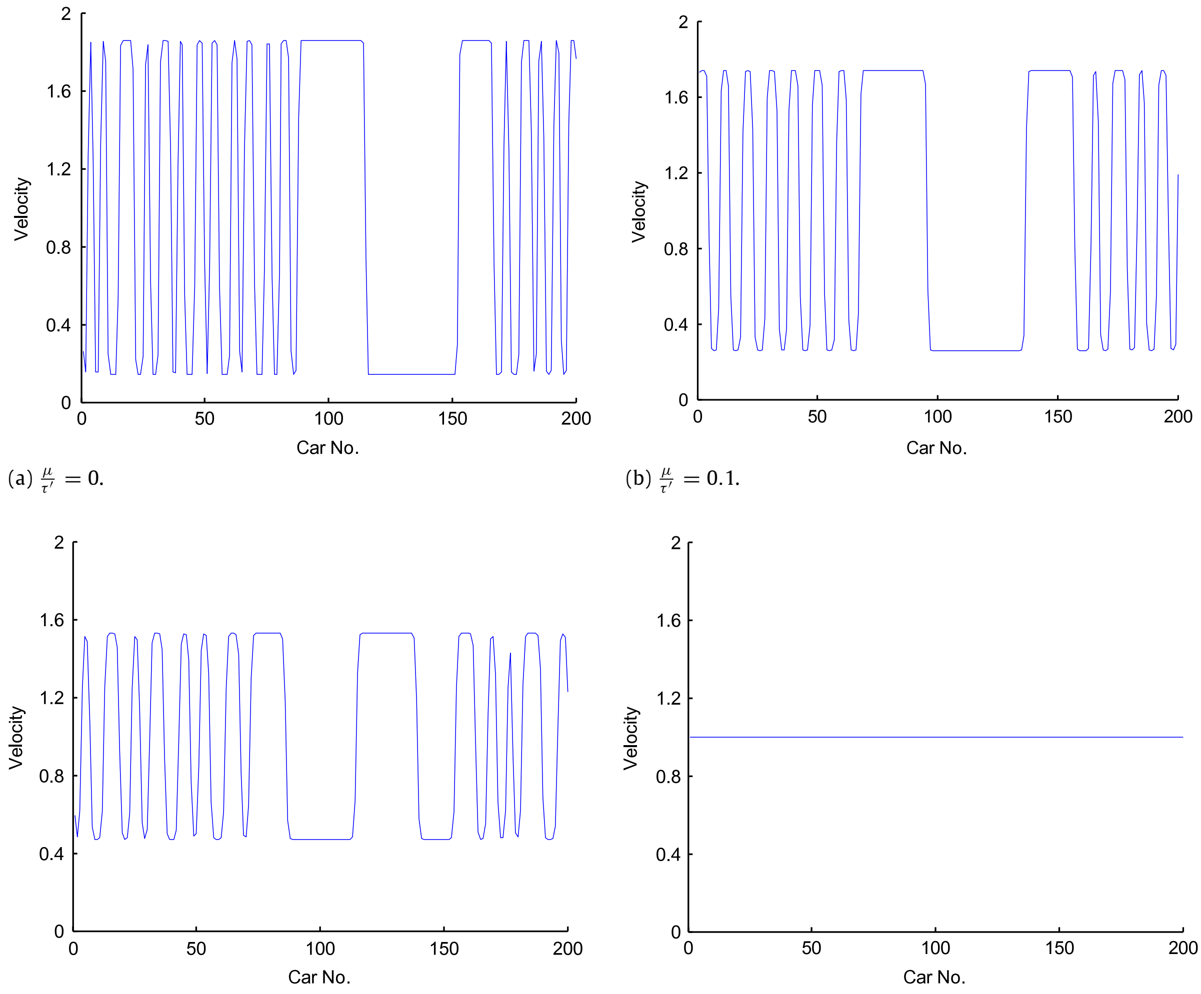
where $N = 200$ is the number of vehicles and $\Delta x_0 = 4.0$ is the average headway. A periodic boundary condition is adopted in the simulation. Other parameters are:

$$v_{\max} = 2.0, \qquad \alpha = 2.0, \qquad L = 800, \tag{21}$$

where L is the length of the studied highway.

Fig. 3 illustrates the headway evolution after 10^4 time steps and the profile at the time step $t = 10^4$, where (a) is the output of the OV model and (b)–(d) are respectively the outputs of our model with $\frac{\mu}{\tau'} = 0.1$, $\frac{\mu}{\tau'} = 0.2$ and $\frac{\mu}{\tau'} = 0.3$. From this figure, we have the following results:

(i) In Fig. 3(a)–(c), stop-and-go traffic appears. This is because the initial value condition lies in the unstable region if the OV model or our model with small parameter $\frac{\mu}{\tau'}$ is used to describe the small perturbations (20). When the small



(a) $\frac{\mu}{\tau'} = 0$.

(b) $\frac{\mu}{\tau'} = 0.1$.

(c) $\frac{\mu}{\tau'} = 0.2$.

(d) $\frac{\mu}{\tau'} = 0.3$.

Fig. 4. Velocity profile at time step $t = 10^4$ for four different $\frac{\mu}{\tau'}$ values.

perturbations are put into uniform flow, they will be amplified with time and the uniform flow will eventually evolve into an inhomogeneous flow. The jam in Fig. 3(a) is the most serious, followed by those in Fig. 3(b) and Fig. 3(c) since the honk effect can improve the stability of traffic flow. The perturbation will finally disappear in Fig. 3(d). This indicates that the introduction of the honk effect can improve the stability of traffic flow.

(ii) Fig. 3(b)–(c) indicate that the small parameter $\frac{\mu}{\tau'}$ cannot completely eliminate the small perturbation though the honk effect can improve the stability of traffic flow, so we should consider the honk effect (the parameter $\frac{\mu}{\tau'}$ is large enough) in the car-following model for further enhancing the stability of traffic flow.

(iii) The density waves in Fig. 3(a)–(d) always propagate backwards. This has been observed in reality and reported in relevant researches.

We next depict the velocity profiles at $t = 10^4$ under the small perturbation (20) (see Fig. 4). Similar results to the points (i) and (ii) stated above can be concluded.

In order to further prove that the honk effect can improve the stability of traffic flow, we now use Eq. (20) to analyze the impact of the honk effect on the hysteresis loop (see Fig. 5). From Fig. 5, we find that the hysteresis loop will gradually be reduced with the increase of the parameter $\frac{\mu}{\tau'}$ and eventually reduced to a point when $\frac{\mu}{\tau'} = 0.3$. This further verifies that the traffic flow stability is indeed improved by the honk effect.

5. Conclusions

In some countries, drivers will honk his/her horn when the leading vehicle prevents him/her from running. In order to describe the honk effect, some models have been proposed and many important results have been obtained, but the impacts of the honk effect on the stability of traffic flow have not been investigated. In this paper, we have proposed an extended car-following model taking the honk effect into account. The analytical and numerical results illustrate that the honk effect can improve the stability of traffic flow and the stability increases with the honk effect coefficient and decreases with the reactive time, which can help us to better understand the impacts of the honk effect on traffic flow and explain some complex

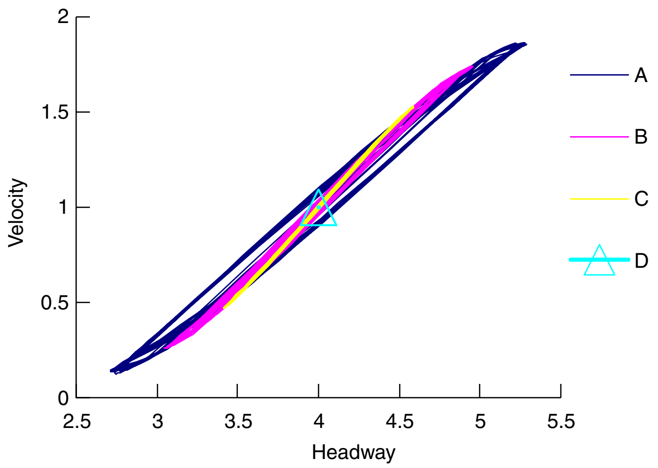


Fig. 5. Hysteresis loops of the OV model and our model, where the parameters in the curves A–D are $\frac{\mu}{\tau'} = 0$, $\frac{\mu}{\tau'} = 0.1$, $\frac{\mu}{\tau'} = 0.2$ and $\frac{\mu}{\tau'} = 0.3$.

phenomena resulted by the honk effect since the honk effect widely appears in many developing countries (e.g. China) although the honk effect seldom occurs in the developed countries. The results obtained in this paper are qualitative, so we will in the future use observed data to further investigate the inherent relationship between the honk effect and the stability of traffic flow.

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