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A New Car-Following Model with Consideration of Driving Resistance *

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Based on the effects of driving resistance on car movement, we develop a new car-following model. The simulation results show that our model can describe the kinetic property of each car during the processes of starting and braking, however the braking process will be postponed and a prominent wavefront will appear during the braking process. With the increase in driving resistance, a car's movement will become more stable during the whole process, the headway of each car will increase and the wavefront will become more prominent. In addition, our model can reproduce the evolution of a small perturbation.

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So far, serious traffic problems have attracted scholars to develop many traffic flow models to study various complex traffic phenomena.[1,2] Among the existing traffic flow models, the cellular automaton (CA) model can describe the micro property of car motion, so it was used to explore some microscopic phenomena of traffic flow.[3–7] In fact, the car-following model can also describe the micro phenomena of traffic flow. Pipe[8] proposed the first car-following model and scholars have extended this model.[9–12] Since the car-following model consists of a series of ordinary differential equations and is restricted by the performance of computers, few car-following models were developed from 1970s to 1980s. With the development of computers, scholars later proposed many new car-following models from different perspectives.[13–28] These car-following models can describe many complex traffic phenomena, but they cannot be employed to explore the effects of driving resistance on traffic flow. In a real traffic system, driving resistance affects car motion. Based on the property of the driving resistance, in this Letter we develop a new car-following model and use it to explore the dynamic property of each car during the processes of the starting and braking and the evolution of a small perturbation.

The car-following model on a single lane can be reduced as follows:[1]

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f\left(v_n, \Delta x_n, \Delta v_n\right), \tag{1}$$

where v_n is the nth car's velocity, $f(\cdot)$ is the stimulus function determined by the headway $\Delta x_n = x_{n+1} - x_n$ (x_n is the nth car's position), the relative velocity $\Delta v_n = v_{n+1} - v_n$ and the velocity v_n (see Fig. 1). In order to further study traffic flow, scholars later developed some extended models to consider multi headways:[15,16,19,20]

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f\left(v_n, \Delta x_n, \Delta x_{n+1}, \cdots, \Delta x_{n+m}, \Delta v_n\right), \tag{2}$$

where $\Delta x_{n+i} = x_{n+i+1} - x_{n+i}$. Zhao and Gao[22] found that collisions will occur under certain conditions when the full velocity difference (FVD) model[17] is used to describe traffic flow, thus they developed a new car-following model which considered the acceleration of the leading car:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f\left(v_n, \Delta x_n, \Delta v_n, \mathrm{d}v_{n+1}/\mathrm{d}t\right). \tag{3}$$

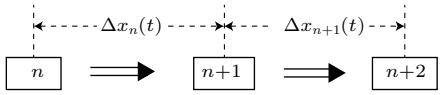

Fig. 1. Scheme of the car-following model.

To further enhance the stability of traffic flow, Wang *et al.*[23] proposed a multiple velocity difference model as follows:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = f\left(v_n, \Delta x_n, \Delta v_n, \Delta v_{n+1}, \cdots, \Delta v_{n+k}\right), \tag{4}$$

where $\Delta v_{n+i} = v_{n+i+1} - v_{n+i}$.

Although the above models can describe some complex phenomena, they cannot directly be used to study the effects of driving resistance on car-following behavior. In fact, each car has a driving resistance that will affect its following behavior, thus Yu[29] pointed out that the drivers should consider driving resistance during the driving process. In a real traffic

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system, the driving resistance of each car is very complex, but we can decompose it into four parts based on Ref. [29]:

$$F'_n = F_n^{\text{rolling}} + F_n^{\text{air}} + F_n^{\text{slope}} + F_n^{\text{acceleration}}, \quad (5)$$

where $F_n^{\text{rolling}}, F_n^{\text{air}}, F_n^{\text{slope}}, F_n^{\text{acceleration}}$ are, respectively, the rolling resistance, air resistance, slope resistance and acceleration resistance of the nth car (see Fig. 2). Here we only study the effects of $F_n^{\text{rolling}}, F_n^{\text{acceleration}}$ on the driving behavior for simplicity, thus Eq. (5) can be rewritten as[29]

$$F'_n = F_n^{\text{rolling}} + F_n^{\text{acceleration}} = m_n g f_n + \delta_n m_n \frac{\mathrm{d}v_n}{\mathrm{d}t}, \quad (6)$$

where m_n is the nth car's mass, g is the gravity acceleration, f_n is the nth car's rolling resistance coefficient and δ_n is the correction coefficient of the rotating mass. Since the FDV model did not consider driving resistance, the resultant force of the FVD model is just the driving force of the nth car. Thus, we can obtain the nth car's driving force from the FVD model,[17] based on the Newton's motion law, i.e.

$$F_n = m_n \frac{dv_n}{dt} = m_n \left[\kappa \left(V\left(\Delta x_n\right) - v_n\right) + \lambda \Delta v_n\right], \quad (7)$$

where κ and λ are the coefficients of the FVD model. The nth car's resultant force is equal to the difference between its driving force and the driving resistance, i.e.

$$F_n^{\text{resultantforce}} = F_n - F'_n. \quad (8)$$

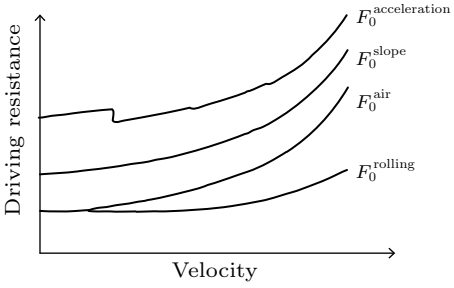

Fig. 2. The relationship between the driving resistance of the nth car and its velocity.[29]

Using Newton's motion law and Eqs. (6)–(8), we can immediately obtain the acceleration of the nth car, i.e.

$$\frac{\mathrm{d}^2 x_n(t)}{\mathrm{d}t^2} = \frac{\kappa\left[V\left(\Delta x_n\right) - v_n(t)\right] + \lambda \Delta v_n - \sigma g f_n}{1 + \delta_n}, \quad (9)$$

where $V\left(\Delta x_n\right)$ is the optimal velocity, κ, λ and σ are three car-following coefficients. Based on Ref. [17] we also set κ as constant and define λ as

$$\lambda = \begin{cases} a, & \Delta x_n \le s_{\mathrm{c}}, \\ b, & \Delta x_n > s_{\mathrm{c}}, \end{cases} \quad (10)$$

where a, b and s_{c} are constants, σ can be defined as

$$\sigma = \begin{cases} 1, & v_n > 0, \\ 0, & v_n = 0. \end{cases} \quad (11)$$

In addition, the optimal velocity $V\left(\Delta x_n\right)$ is defined as[14]

$$V(\Delta x_n) = V_1 + V_2 \tanh\left[C_1(\Delta x_n - l_c) - C_2\right], \quad (12)$$

where l_{c} is the car length, V_1, V_2, C_1, C_2 are constants.

Based on Ref. [29] we can approximately define the parameter δ_n as 1. The parameter f_n is often relevant to the nth car's performance and the quality of the road, but here we define it as a constant for simplicity. The parameters in Eqs. (7)–(12) have no qualitative effects on the following numerical results, so we define them as follows:

$$\kappa = 0.41\ \text{s}^{-1},\ g = 9.8\ \text{m/s}^2,\ a = 0.5,\ b = 0,$$
$$s_{\mathrm{c}} = 150\ \text{m},\ V_1 = 6.75\ \text{m/s},\ V_2 = 7.91\ \text{m/s},$$
$$C_1 = 0.13\ \text{m}^{-1},\ C_2 = 1.57, l_c = 5\ \text{m}.$$

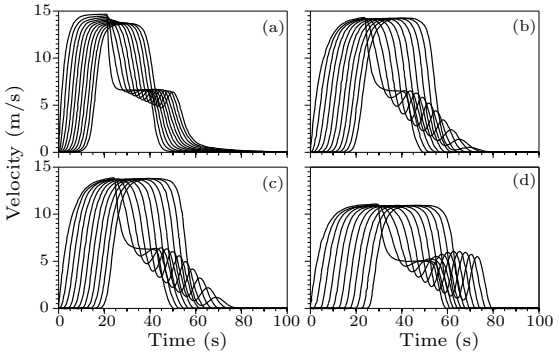

Fig. 3. The velocity evolution of each car: (a) FVD model, (b) $f = 0.01$, (c) $f = 0.03$, (d) $f = 0.15$.

In order to compare our model with the FVD model, the simulation conditions are set as: 11 cars distribute on a road, where the 11th car is at the origin and the headway of other cars is 7.4 m; there is a traffic light and a barrier on the road, where the traffic light is located at 74 m and the barrier is located at 500 m; all cars are still when $t < 0$; the signal light turns green and all cars will immediately start at $t = 0$; all cars will eventually stop. First, we study the evolution of the velocity of each car during the processes of the starting and braking (see Fig. 3). From this figure, we can conclude the following results: (1) Compared to the FVD model, the velocity of our model is less than that of the FVD model because the driving resistance will reduce the acceleration of each car. In addition, the reduction of the velocity will increase with the driving resistance. (2) During the braking process, the FVD model will produce three prominent phases, i.e. the velocity of each car will sharply drop in the first phase, the velocity will sharply drop again after a short fluctuation in the second phase and the

velocity will gradually drop to zero in the third phase; our model will produce the above three phases, too, but the drop trend of the velocity is more smooth, the fluctuation amplitude increases with the driving resistance and the driving resistance makes the second phase more prominent.

Using Fig. 3, we further study the delay time δt and the kinematic wave velocity c_{j} at jam density, δt, c_{j} are often complex in the braking process and they should be demarcated by observed data, so we only list them in the starting process in this study (see Table 1). From Table 1, we find that δt will increase and c_{j} will decrease with the driving resistance.

Table 1. Delay time and kinematic wave velocity at jam density in the process of starting. FISR: frozen or impacted snow road.

Parameters	Suitable situations	δt (s)	c_j (km/h)
FVD model	Ideal road	1.4	19.03
$f = 0.01$	Asphalt or beton road	2.0	13.32
$f = 0.03$	FISR	2.1	12.69
$f = 0.15$	Muddy road	2.6	10.25

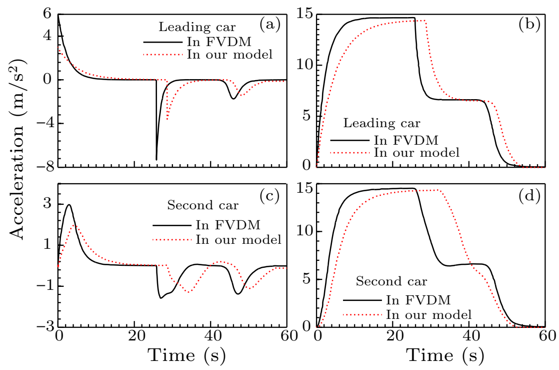

Fig. 4. The velocity and acceleration of the leading car and its following car, where $f = 0.01$ in our model.

Next, we investigate the evolution of the velocity and acceleration of the leading car and its following car (see Fig. 4) and the motion trail of each car (see Fig. 5) during the starting and braking processes. From the two figures, we can conclude the following results: (a) During the starting process, the acceleration of our model is first lower then higher than that of the FVD model, but the two accelerations will eventually evolve into zero (see Figs. 4(a) and 4(c)); the velocity of our model is more stable than that of the FVD model (see Figs. 4(b) and 4(d)). The reasons are as follows: the cars of our model will take longer time to reach the maximum velocity than that of the FVD model, so they will still accelerate when the cars of the FVD model have already reached the maximum velocity, which leads to the acceleration of our model higher than that of the FVD model during this time; but the cars of the two models will eventually accelerate to the maximum velocity, so the accelerations of the two models will evolve into zero. (b) The driving resistance has reduced the velocity, so the cars of our model will take longer time to reach the barrier than the FVD model, i.e. the braking process of our model is delayed (see Fig. 4). (c) All cars will quickly evolve into the uniform motion during the starting process; remarkable wavefronts will appear during the braking process; these wavefronts will turn more prominent and the headway of each car will gradually increase with the driving resistance (Fig. 5).

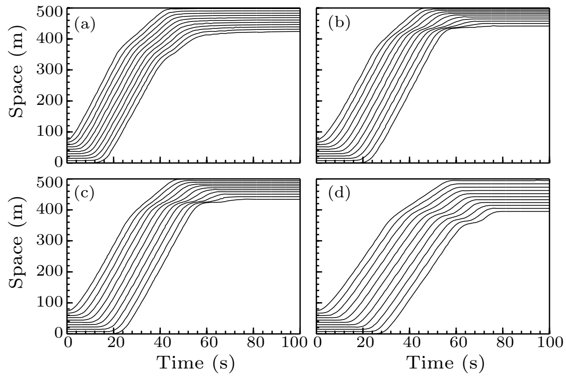

Fig. 5. The trail of each car. (a) FVD model; (b) $f = 0.01$; (c) $f = 0.03$; (d) $f = 0.15$.

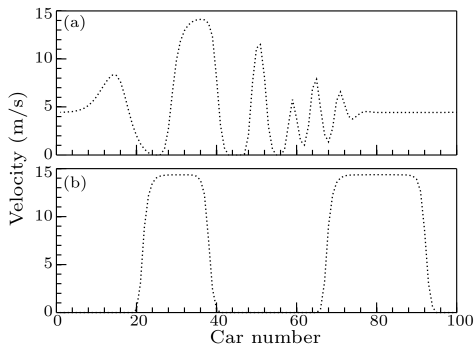

Fig. 6. The velocity of each car under the small perturbation, where (a) $t = 150$ s and (b) $t = 1500$ s.

Finally, we adopt periodic boundary condition to study the effect of driving resistance on the evolution of small perturbation. The initial conditions are as follows:

$$x_1(0) = 1\ \mathrm{m}; \quad x_n(0) = (n-1)L/n, \quad \text{if } n \neq 1;$$
$$v_n(0) = V(L/n),$$

where $N = 100$, $L = 1500$ m. We here set $\lambda = 0.5\ \mathrm{s}^{-1}$ since the headway is very small and other parameters are the same as that of Fig. 3(b). Figure 6 is the velocity profile of each car at $t = 150$ s and $t = 1500$ s. From Fig. 6, we have: (1) The above perturbation will produce traffic jam phenomenon. (2) There are eventually two types of velocities in the system (Fig. 6(b)): one is approximately equal to zero; the other is approximately equal to maximum velocity. Furthermore, the two types of velocities will alternates, so the stop-and-go phenomenon will appear.

Although many traffic flow models have been developed to describe various complex traffic phenomena, they cannot reproduce the effects of driving resistance on traffic flow. In this study, we present a new car-following model which considers driving resistance. The numerical results show that our model can describe the dynamic property of each car during the processes of starting and braking. However, we assume that the driving resistant is irrelevant to the velocity. In fact, driving resistance is often very complex and depends on many factors (the velocity, the road quality), so we should further study the quantitative relation between the driving resistance and the velocity by use of many experimental date and develop a more reasonable traffic flow model to describe the effects of driving resistant on traffic flow.

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