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An Extended Optimal Velocity Model with Consideration of Honk Effect*

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Abstract *Based on the OV (optimal velocity) model, we in this paper present an extended OV model with the consideration of the honk effect. The analytical and numerical results illustrate that the honk effect can improve the velocity and flow of uniform flow but that the increments are relevant to the density.*

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1 Introduction

So far, scholars have developed many traffic flow models to describe various complex traffic phenomena,[1−19] but they cannot be used to study the honk effect because they did not explicitly consider this factor. Driver will often honk the horn when his leading car prevents him from running at his current velocity, so the honk effect should be considered when analyzing the car-following behavior. In order to study the honk effect, Jia *et al.*[20] proposed a new cellular automaton model and Tang *et al.*[21] presented a bicycle-following model and a pedestrian-following model with the consideration of the honk effects, but they did not investigate the impacts of the honk effect on the car-following behavior. Based on the optimal velocity (OV) model, we in this paper develop an extended OV model with the consideration of the honk effect.

2 Car-Following Model

Among the existing car-following models, the simple and useful one is the OV model,[11] which can be reduced as follows:

$$\frac{dx_n(t+\tau)}{dt} = V(\Delta x_n(t))\,, \tag{1}$$

where $x_n(t)$, $V(\Delta x_n(t))$ are respectively the n-th car's position and optimal velocity at time t, $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ is the gap between the n-th car and its adjacent leading car and τ is the reactive time. In order to further study traffic flow, scholars later presented some extended OV models,[15−16] i.e.

$$\frac{dx_n(t+\tau)}{dt} = V(\Delta x_n(t), \Delta x_{n-1}(t), \ldots, \Delta x_{n-m-1}(t))\,, \tag{2}$$

where $\Delta x_{n-i}(t) = x_{n-i-1}(t) - x_{n-i}(t)$.

The above models can reproduce some complex traffic phenomena, but they cannot be used to describe the honk effect because they did not explicitly consider this factor. In fact, when one car prevents its following car from running at its current velocity, the following car will often honk its horn. When one driver hears his following car's horn, he will enter his adjacent lane or accelerate if possible in the multi-lane system and he will accelerate if possible in the single-lane system, so we should consider the honk effect when we analyze the car-following behavior. Thus, we obtain a new car-following model for single-lane traffic flow based on Eq. (1), i.e.

$$\frac{dx_n(t+\tau)}{dt} = V(\Delta x_n(t), g(v_n(t)))\,, \tag{3}$$

where $V(\Delta x_n(t), g(v_n(t)))$ is the n-th car's optimal velocity at time t,[‡] τ is the delay time and $g(v_n(t))$ is the honk effect resulted by the $(n+1)$-th car. Applying the similar method,[22] we suppose that $V(\Delta x_n, g(v_n))$ can be formulated as a linear combination of the perceived head-induced optimal velocity and the honk effect, i.e.

$$V(\Delta x_n, g(v_n)) = V(\Delta x_n) + \mu g(v_n)\,, \tag{4}$$

where μ is the coefficient that represents the weight of the honk effect, and $V(\Delta x_n)$ is the perceived head-induced optimal velocity and can be defined as follows:[11]

$$V(\Delta x_n) = \frac{v_{\max}}{2}[\tanh(\Delta x_n - h_c) + \tanh(h_c)]\,, \tag{5}$$

where $v_{\max}$ is the maximum velocity and h_c is the safe distance.

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‡Note: the optimal velocity in Eq. (3) is different from the one defined in the OV model.[11]

Expanding the left hand side of Eq. (3) in the Taylor series, neglecting the non-linear order terms and combining Eq. (4), we have

$$\frac{dv_n(t)}{dt} = \frac{V(\Delta x_n) - v_n}{\tau} + \lambda \eta_n g(v_n)\,, \tag{6}$$

where $\lambda = \mu/\tau$ is the honk effect coefficient, and η_n is the honk desire coefficient that the $(n+1)$-th car honks its horn.[§] In fact, $g(v_n)$ is irrelevant to the $(n+1)$-th car's current status but relevant to its current status since the driver cannot adjust his acceleration based on the $(n+1)$-th car's current status but will adjust his acceleration based on his current status. In the single-lane system, the driver will often try to accelerate to the maximum velocity based on his current velocity if possible when he hears his following car's horn, so $g(v_n)$ is relevant to the maximum velocity $v_{\max}$ and the current velocity v_n. Since the exact definition of $g(v_n)$ should be calibrated by use of many experimental data but will have no qualitative effects on the following results, we for simplicity define it as follows:

$$g(v_n) = \frac{v_{\max} - v_n}{\tau'}\,, \tag{7}$$

where τ' is the time that it takes the n-th car to adjust its acceleration based on the honk effect.[¶] Thus Eq. (6) can be rewritten as follows:

$$\frac{dv_n(t)}{dt} = \frac{V(\Delta x_n) - v_n}{\tau} + \lambda \eta_n \frac{v_{\max} - v_n}{\tau'}\,, \tag{8}$$

where the honk desire coefficient η_n will satisfy the following conditions: when Δx_n is very small, there is no prominent effect even if the $(n+1)$-th car honks its horn since the n-th car has no room to accelerate, therefore, the $(n+1)$-th car has no desire to honk its horn; when Δx_n is very large, there is no prominent effect even if the $(n+1)$-th car honks its horn since the n-th car velocity is equal to its maximum velocity, therefore, the $(n+1)$-th car has little desire to honk its horn; when Δx_n is moderate, the $(n+1)$-th car has often desire to honk its horn and the desire that it honks its horn will first increase then decrease with the headway Δx_n. The honk desire coefficient η_n has no qualitative effects on the following results, so we for simplicity define it as follows:

$$\eta_n = \begin{cases} 0\,, & \text{if } \Delta x_n \le h_{1c}\,, \\ -\dfrac{(\Delta x_n - h_{2c})^2}{(h_{1c} - h_{2c})^2} + \eta_{\max}\,, & \text{if } h_{1c} < \Delta x_n \le h_{2c}\,, \\ -\dfrac{(\Delta x_n - h_{2c})^2}{(h_{3c} - h_{2c})^2} + \eta_{\max}\,, & \text{if } h_{2c} < \Delta x_n \le h_{3c}\,, \\ 0\,, & \text{if } \Delta x_n \ge h_{3c}\,, \end{cases} \tag{9}$$

where h_{1c}, h_{2c}, h_{3c} are three critical headways, and $\eta_{\max}$ is the maximum probability that the $(n+1)$-th car honks its horn.

3 Simulation

In this section, we investigate the impacts of the honk effect on uniform flow. As for uniform flow, each car's acceleration is equal to zero, the gap between two adjacent cars is equal to the mean gap h and the honk desire coefficient η_n is equal to a constant (here we set it as η_0), thus Eq. (8) can be rewritten as follows:

$$\frac{V(h) - v_n}{\tau} + \lambda \eta_0 \frac{v_{\max} - v_n}{\tau'} = 0\,. \tag{10}$$

Based on the relationship between the density and headway, the car-following model's density can be defined as follows:

$$\rho = \frac{1}{\Delta \bar{x}}\,, \tag{11}$$

where $\Delta\bar{x}$ is the headway, where the headway is equal to the summation of the gap between two adjacent cars and the length of a car, i.e.

$$\Delta \bar{x} = \Delta x + l\,, \tag{12}$$

where Δx, l are respectively the mean gap between two adjacent cars and the car length. Combining with Eq. (12), Eq. (5) can be rewritten as follows:

$$V(\rho) = \frac{v_f}{2}\Big[\tanh\Big(\frac{1}{\rho} - \frac{1}{\rho_c}\Big) + \tanh\Big(\frac{1}{\rho_c} - l\Big)\Big]\,, \tag{13}$$

where v_f is the free flow velocity and ρ_c is the critical density, which can be defined as follows:

$$\rho_c = 1/(h_c + l)\,, \tag{14}$$

where h_c is the critical gap between two adjacent cars. Thus, our model's velocity under uniform flow can be reduced as follows:

$$v = \frac{\tau' V(\rho) + \lambda \eta_0 \tau v_f}{\lambda \eta_0 \tau + \tau'} \ge V(\rho)\,. \tag{15}$$

It is easy to prove that Eq. (15) will increase with the density and that our model's velocity is greater than the OV model's. Our model's flow under uniform flow can be reduced as follows:

$$q = \rho\Big(\frac{\tau' V(\rho) + \lambda \eta_0 \tau v_f}{\lambda \eta_0 \tau + \tau'}\Big) = \frac{\tau'}{\lambda \eta_0 \tau + \tau'}(\rho V(\rho)) + \frac{\lambda \eta_0 \tau}{\lambda \eta_0 \tau + \tau'}(\rho v_f) \ge \rho V(\rho)\,. \tag{16}$$

The parameters in Eqs. (13)–(16) have no qualitative impacts on the following results, so we here define them as follows: $v_f = 2$ m/s, $h_c = 4$ m, $l = 1$ m, $\lambda = 0.1$, $\tau = 2$ s, $\tau' = 5$ s, $h_{1c} = 1$ m, $h_{2c} = 9$ m, $h_{3c} = 19$ m, $\eta_{\max} = 1$.

Using the above parameters, we can obtain the velocity-density and flow-density relationships of uniform flow (see Fig. 1). From the figure, we can conclude the following results:

[§] For simplicity, we here assume that the driver of the n-th car will immediately accelerate if possible once he hears the $(n+1)$-th car's horn.

[¶] Note: τ' is often greater than the delay time τ.

(i) The velocity will decrease with density, the flow will first increase then decrease with density;

(ii) When the density is very low or very high, there is little difference between our model's velocity and flow and the OV model's; our model will enhance the velocity and flow when the density is moderate, which are qualitatively accordant with the real traffic.

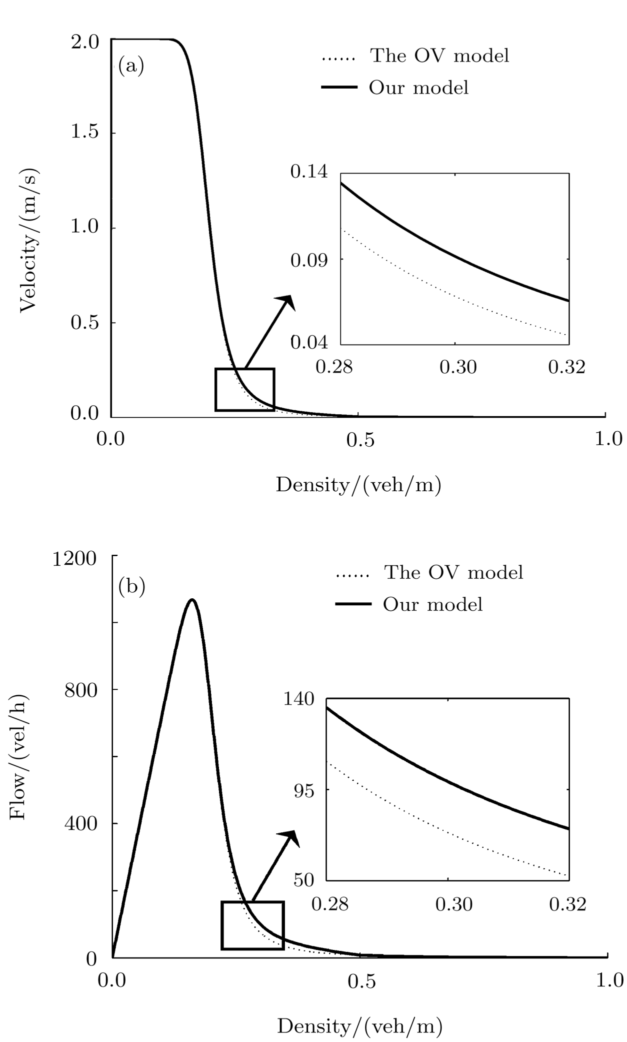

Fig. 1 The relationship among the velocity, flow and density of uniform flow, where (a) is the velocity-density relationship and (b) is the flow-density relationship.

In order to further describe the quantitative relationship between the honk effect and the density, we study the difference between our model's velocity and the OV model's and the difference between our model's flow and the OV model's, where the two differences are reduced as follows:

$$\frac{\tau' V(\rho)+\lambda\eta_0\tau v_f}{\lambda\eta_0\tau+\tau'}-V(\rho)=\frac{\lambda\eta_0\tau}{\lambda\eta_0\tau+\tau'}(v_f-V(\rho))\,,\quad(17)$$

$$\rho\Big(\frac{\tau' V(\rho)+\lambda\eta_0\tau v_f}{\lambda\eta_0\tau+\tau'}\Big)-\rho V(\rho)=\frac{\lambda\eta_0\tau}{\lambda\eta_0\tau+\tau'}(\rho v_f-\rho V(\rho)).\quad(18)$$

Using Eqs. (9), (13), (17), (18) and the above parameters, we find that the velocity and flow differences first increase then decrease with the density when the density is moderate and the velocity and flow differences is equal to zero when the density is very low or very high (see Fig. 2).

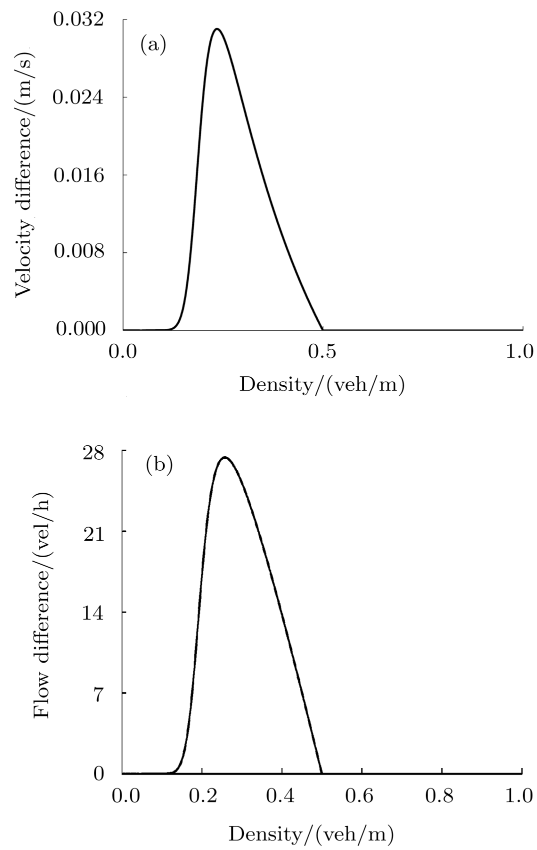

Fig. 2 The differences between our model's velocity and flow and the OV model's velocity and flow, where (a) is the velocity difference and (b) is the flow difference.

Finally, we study the sensitivities of the two essential parameters of the honk effect. Set

$$v(\lambda,\tau')=\frac{\tau' V(\rho)+\lambda\eta_0\tau v_f}{\lambda\eta_0\tau+\tau'}\,,$$

$$q(\lambda,\tau')=\rho\frac{\tau' V(\rho)+\lambda\eta_0\tau v_f}{\lambda\eta_0\tau+\tau'}\,,$$

thus we have

$$\frac{\partial v(\lambda,\tau')}{\partial\lambda}=\frac{\eta_0\tau\tau'(v_f-V(\rho))}{(\lambda\eta_0\tau+\tau')^2}\geq 0\,,\quad(19)$$

$$\frac{\partial v(\lambda,\tau')}{\partial\tau'}=\frac{\lambda\eta_0\tau(V(\rho)-v_f)}{(\lambda\eta_0\tau+\tau')^2}\leq 0\,,\quad(20)$$

$$\frac{\partial q(\lambda,\tau')}{\partial\lambda}=\rho\frac{\eta_0\tau\tau'(v_f-V(\rho))}{(\lambda\eta_0\tau+\tau')^2}\geq 0\,,\quad(21)$$

$$\frac{\partial q(\lambda,\tau')}{\partial\tau'}=\rho\frac{\lambda\eta_0\tau(V(\rho)-v_f)}{(\lambda\eta_0\tau+\tau')^2}\leq 0\,.\quad(22)$$

From Eqs. (19)–(22), we can conclude that our model's velocity and flow will increase with the honk effect coefficient λ and decrease with the time τ'.

Using the above parameters, we can obtain the relationship between the velocity and the density and the relationship between the flow and the density under difference parameters λ, τ' (see Figs. 3–6). From the four figures, we find that our model's velocity and flow will increase with the parameter λ and decrease with the parameter τ', which is accordant with the analytical results.

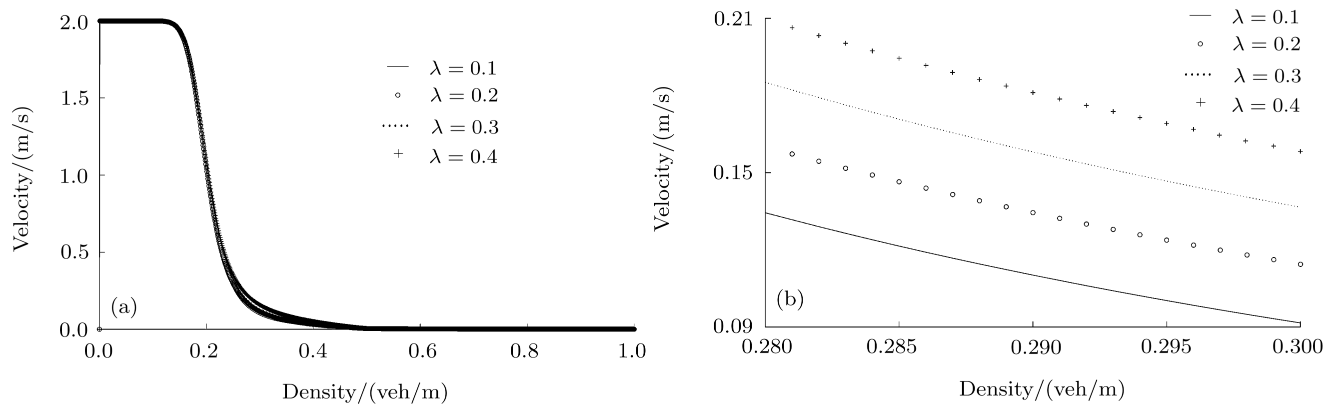

Fig. 3 The velocity-density curve of uniform flow with different parameter λ, where (a) is the whole curve and (b) is the local amplified one of (a).

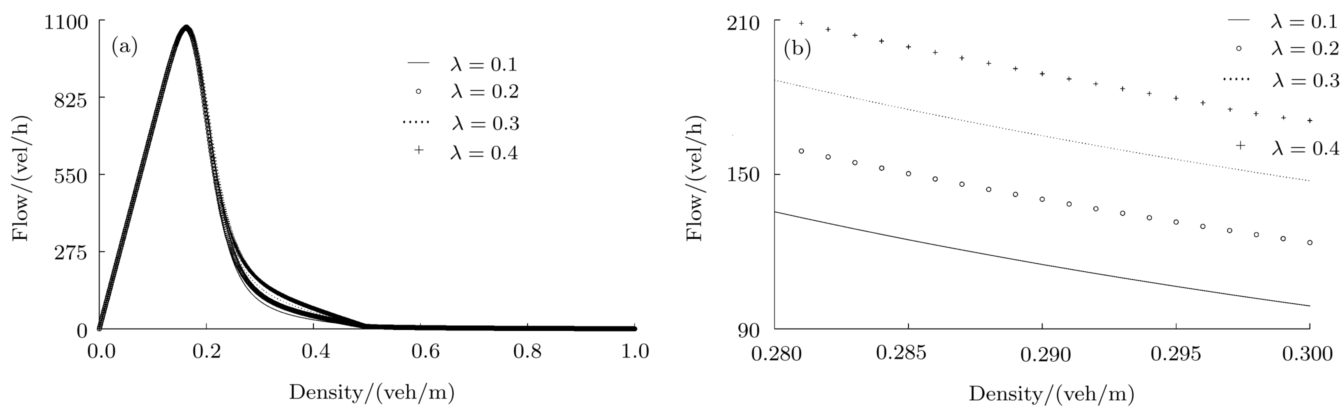

Fig. 4 The flow-density curve of uniform flow with different parameter λ, where (a) is the whole curve and (b) is the local amplified one of (a).

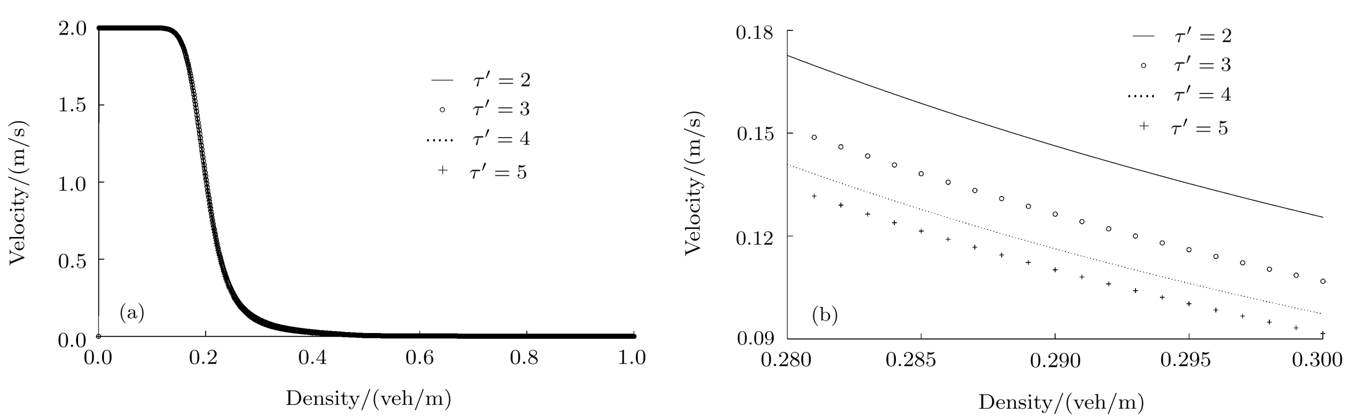

Fig. 5 The velocity-density curve of uniform flow with different parameter τ', where (a) is the whole curve and (b) is the local amplified one of (a).

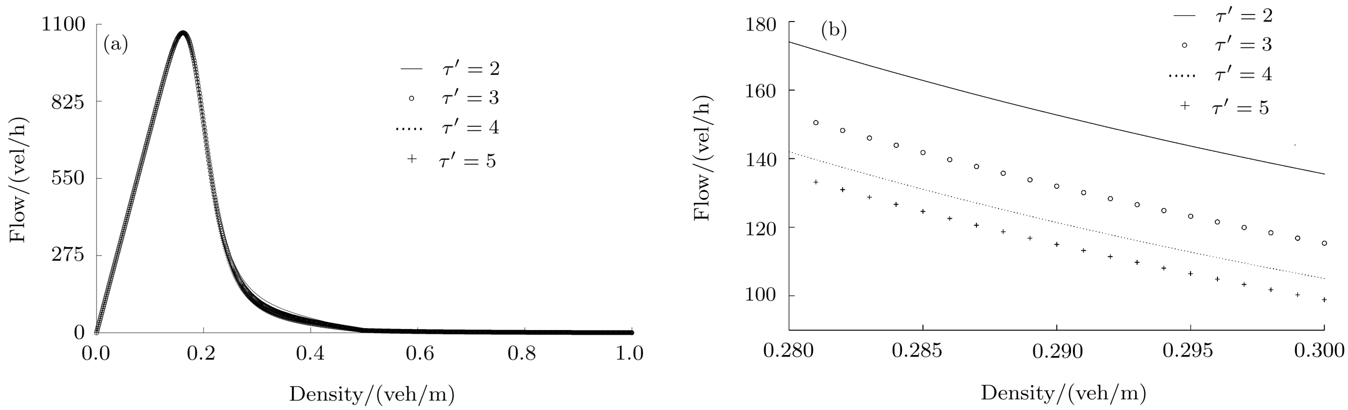

Fig. 6 The flow-density curve of uniform flow with different parameter τ', where (a) is the whole curve and (b) is the local amplified one of (a).

4 Conclusions

Many traffic flow models have been developed to describe various complex traffic phenomena, but the existing models cannot directly be used to study some complex phenomena resulted by the honk effect since they did not explicitly consider this factor. In this paper, we present a new car-following model with the consideration of the honk effect based on the OV model. The analytical and numerical results show that our model can enhance the velocity and flow of uniform flow but the increments are relevant to the density, so our model can be used to describe the complex traffic phenomena resulted by the honk effect. However, we only study the impacts of the honk effect on uniform flow. In fact, the honk effect is very complex and depends on many factors, so we should in the future use many observed data to further study the quantitative relationships among the honk effect and the traffic flow parameters (e.g. velocity, flow, and density) and develop an exact model to describe various complex phenomena resulted by the honk effect. In addition, the honk effect is not very prominent even if the numerical results that are shown in Figs. 1–6 illustrate that the honk effect can improve the velocity and flow. The main reasons are that we in this paper analyze the honk effect in the single-lane traffic system, so we should in the future extend this car-following model to the multi-lane mixed traffic system and further study the impacts of the honk effect on the multi-lane mixed traffic system.

References

[1] D. Chowdhury, L. Santen, and A. Schreckenberg, Phys. Rep. **329** (2000) 199.

[2] D. Helbing, Rev. Mod. Phys. **73** (2001) 1067.

[3] R. Jiang, Q.S. Wu, and Z.J. Zhu, Transp. Res. B **36** (2002) 405.

[4] M.J. Lighthill and G.B. Whitham, Proc. R. Soc. London – Ser. A **229** (1955) 317.

[5] P.I. Richards, Oper. Res. **4** (1956) 42.

[6] K.P. Li and Z.Y. Gao, Chin. Phys. Lett. **21** (2004) 2120.

[7] X.M. Sun and Y.J. Dong, Chin. Phys. Lett. **23** (2006) 2494.

[8] X.Y. Zhu, Z.H. Li, and M. Tang, Chin. Phys. Lett. **24** (2007) 2142.

[9] M.L. Yang, Y.G. Liu, and Z.S. You, Chin. Phys. Lett. **24** (2007) 2910.

[10] T. Chen, B. Jia, X.G. Li, R. Jiang, and Z.Y. Gao, Chin. Phys. Lett. **25** (2008) 2795.

[11] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, Phys. Rev. E **51** (1995) 1035.

[12] G.F. Newell, Oper. Res. **9** (1961) 209.

[13] D. Helbing and B. Tilch, Phys. Rev. E **58** (1998) 133.

[14] R. Jiang, Q.S. Wu, and Z.J. Zhu, Phys. Rev. E **64** (2001) 017101.

[15] H.X. Ge, S.Q. Dai, L.Y. Dong, and Y. Xue, Phys. Rev. E **70** (2004) 066134.

[16] T.Q. Tang, H.J. Huang, S.G. Zhao, and G. Xu, Int. J. Mod. Phys. B **23** (2009) 743.

[17] T.Q. Tang, H.J. Huang, X.Y. Xu, and Y. Xue, Chin. Phys. Lett. **24** (2007) 1410.

[18] X.M. Zhao and Z.Y. Gao, Eur. Phys. J. B **47** (2005) 145.

[19] T. Wang, Z.Y. Gao, and X.M. Zhao, Acta Phys. Sin. **55** (2006) 634.

[20] B. Jia, R. Jiang, Q.S. Wu, and M.B. Hu, Physica A **348** (2005) 544.

[21] T.Q. Tang, H.J. Huang, and H.Y. Shang, Int. J. Mod. Phys. B, in press.

[22] T.Q. Tang, H.J. Huang, S.C. Wong, and R. Jiang, Acta Mech. Sin. **24** (2008) 399.